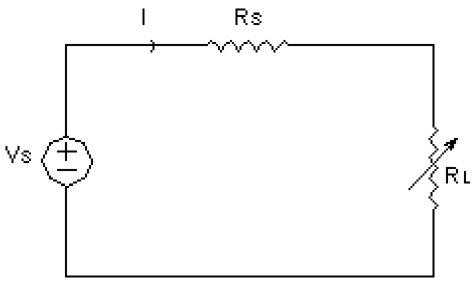
CONDITION FOR

MAXIMUM POWER AND **MAXIMUM REACTIVE POWER**

I. CONDITION FOR MAXIMUM POWER:

The maximum power transfer theorem states that maximum power will be delivered to that load resistance R_L for which $R_L = R_S$ driven by an independent voltage source having a series resistance R_S.

Let us consider a circuit given below:



For this circuit the power absorbed by the load resistance 'R_L' is, $P_L = I^2 R_L$
where $I = V_S / (R_S+R_L)$

$$\therefore P_L = \left(\frac{V_S}{R_S + R_L}\right)^2 R_L = V_S^2 \cdot \frac{R_L}{(R_S + R_L)^2}$$

To get the condition for this power to be maximum, differentiate P_L w.r.t. 'R_L' and equate it to zero.

$$i.e. \frac{dP_L}{dR_L} = 0 \Rightarrow \frac{d}{dR_L}\left[V_S^2 \frac{R_L}{(R_S + R_L)^2}\right] = 0$$

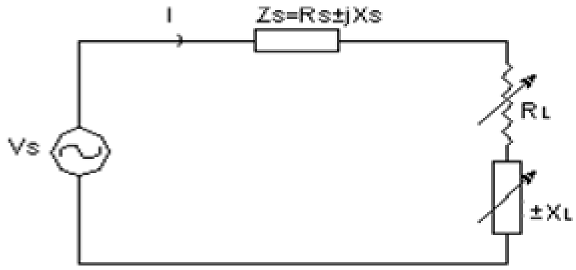
$$\Rightarrow \frac{V_S^2\left[(R_S + R_L)^2.1 - R_L.2\,(R_S + R_L)\right]}{(R_S + R_L)^4} = 0$$

$$\Rightarrow \left[(R_S + R_L)^2.1 - R_L.2\,(R_S + R_L)\right] = 0$$

$$\Rightarrow (R_S + R_L) - 2R_L = 0$$

$$\boxed{R_L = R_S} \quad and\ P_{L\max} = \frac{V_S^2}{4R_S} \quad \text{watts.}$$

Now consider the most interesting *A/c circuit* given below:



Zs = Source internal impedance;
Rs = source internal resistance;
Xs = source internal reactance and it could be *either inductive reactance or capacitive reactance*;
Z_L = Load impedance;
R_L = Load resistance;
X_L = Load reactance and it could be *either load inductive reactance or load capacitive reactance*;

To obtain the condition for power transfer, let us analyze the following cases:

Case-1: If both R_L and X_L are variable.

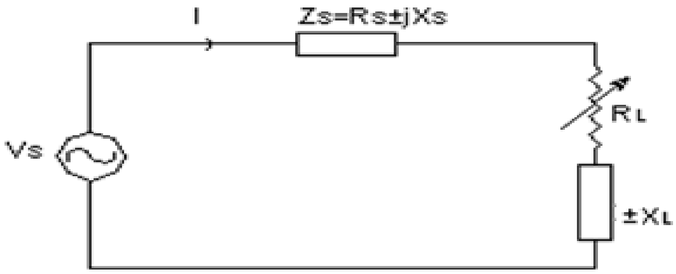
We know that $P_L = I^2 . R_L$

By looking at the given circuit one can easily confirm that the condition for maximum P_L is

$\boxed{R_L = R_S}$ and $\boxed{X_L = X_S^*}$ i.e., X_L is the conjugate of X_S.

Thus, if $\boxed{Z_L = Z_S^*}$ i.e., if Z_L is the complex conjugate of Z_S then, the power transferred to the load will be maximum.

Case-2: If only R_L is variable and X_L is constant.



$$P_L = I^2 R_L = \left(\frac{V_S}{Z}\right)^2 . R_L; \quad \text{where } \bar{Z} = \bar{R}_L + \bar{Z}_1; \text{ and } Z_1 = \left(R_S + j\left(X_S + X_L\right)\right)$$

Proof:

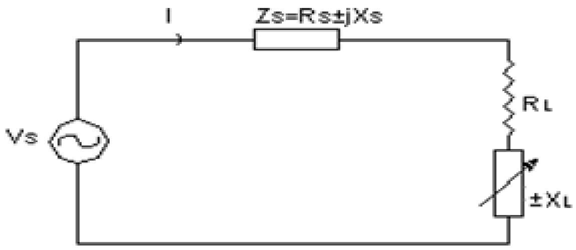
$$P_L = \left(\frac{V_S}{Z}\right)^2 R_L = V_S^2 \left(\frac{R_L}{\left(R_L + Z_1\right)^2}\right).$$

$$\therefore \frac{dP_L}{dR_L} = 0 \Rightarrow \frac{V_S^2 \left[\left(R_L + Z_1\right)^2 .1 - R_L.2\left(R_L + Z_1\right)\right]}{\left(R_L + Z_1\right)^4} = 0$$

$$\Rightarrow (R_L + Z_1)^2 - 2R_L\,(R_L + Z_1) = 0 \qquad \Rightarrow (R_L + Z_1) = 2R_L \Rightarrow R_L = |Z_1|.$$

Therefore, the condition for P_L max, $\boxed{R_L = |Z_1|}$.

Case-3: Only X_L is variable and R_L is constant



$$P_L = I^2\,R_L \quad where\, I = \frac{V_S}{(R_S + R_L) + j\,(X_S + X_L)}.$$

Then, P_L will be max, when I is max. i.e. when $\boxed{X_L = X_S^{\,*}}$ for a given R_L.

II. CONDITION FOR MAXIMUM REACTIVE POWER:

For the above circuit the reactive power consumed by the load is given by $Q_L = I^2.X_L$.

$$where\, I = \frac{V_S}{(R_S + R_L) + j\,(X_S + X_L)}$$

$$\therefore Q_L = \left(\frac{V_S}{Z}\right)^2 X_L;\;\; Z = Z_1 + jX_L;\;\; where\; Z_1 = ((R_S + R_L) + jX_S)$$

The condition for Q_L max. is obtained by calculating $\frac{dQ_L}{dX_L} = 0$, which yields $\boxed{X_L = |Z_1|}$ **with the sign of the X_L opposite to that of X_S.**

As an example, with V_s = 400 V; R_s = 30 ohms; X_s = 20 ohms (inductive reactance); R_L = 25 ohms; the values obtained are **$Q_{L\ max}$ = - 2076.65 Reactive VA (capacitive)** and the corresponding **X_L = - 58.52 ohms (capacitive reactance).**